Equations (27)–(34) show that the changes in thermal conductivity and ultrasonic attenuation are simple functions of the parameter $\mu = 2\sqrt{\pi}(\Delta/k_c v_F)^2 k_c l$ and are, therefore, strongly mean-free-path dependent. In Figs. 1–3, we plot $(\Delta\kappa/\kappa_n)^2$, $(\Delta\alpha^T/\alpha_n)^2$, and $(\Delta\alpha^L/\alpha_n)^2$, respectively, as functions of μ. As is easily seen from the figures, the relative change in the square of the transport coefficients for fixed l can be considered as proportional to Δ^2, that is, $H_{c2} - B$, over a narrow field range close to H_{c2}. Both of these effects are consistent with experiment. Further, we note that the theory correctly predicts the experimentally observed anisotropy in the thermal conductivity near H_{c2}.

Finally, we should point out that we only expect the theory to be valid for $\mu < 1$: For values of $\mu > 1$, that is, for fields $H \ll H_{c2}$ and/or purer samples, it will be necessary to determine the single-particle propagator used in this theory to a higher degree of accuracy.

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Transport Properties of Clean Type-II Superconductors in the Flux-Flow Regime

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We recalculate the flux-flow resistivity and the Ettingshausen coefficient of clean type-II superconductors in the high-field region. Use is made of a Green's function due to Brandt, Pesch, and Tewordt, which enables accurate calculation of the transport properties in the vicinity of the upper critical field. Both the flux-flow resistivity and the Ettingshausen effect can be compared with the previous calculation in the limit of the small order parameter, although the present expression for the flux-flow resistivity results in a slope at $H = H_{c2}$ larger by a factor of 2. We also find a strong mean-free-path dependence of these coefficients in lower fields.

I. INTRODUCTION

In recent years there has been a great deal of work, both theoretical and experimental, on the transport properties of type-II superconductors[1,2] in the flux-flow regime. The dynamical properties (e.g., flux-flow resistivity and Ettingshausen effect) in the flux-flow regime are of particular interest, since they provide invaluable information on the dynamical behavior of the superconducting order parameter (i.e., the way the order parameter in the vortex state moves in response to an electric field or to a temperature gradient).

This phenomenon can be treated from a microscopic point of view, if we limit ourselves to the vicinity of the upper critical field H_{c2}, where the order parameter $\Delta(\vec{r}, t)$ is small. In fact, making use of a perturbation expansion, where we take the order parameter as a small parameter, various transport coefficients have been calculated for both dirty and clean type-II superconductors.[3-7] In dirty type-II superconductors (which are typical gapless superconductors), the above approach should work well, and the theoretical calculation can explain, for the most part, all of the transport properties[1-3] (with the possible exception of the Hall current) so far observed experimentally. On the other hand, in clean type-II superconductors, we are not sure beforehand whether or not this type of perturbation approach will result in reliable predictions.[8] Previous calculations[7] along these lines by one of us (K. M.) appear to account for at best only the qualitative features of the transport properties[1,2] (e.g., flux-flow resistivity and Ettingshausen effect) of pure Nb samples. In particular, the predicted change in the resistivity in the vortex state appears to be roughly one order of magnitude smaller than that observed.[9-11] Furthermore, these transport coefficients display a strong mean-free-path dependence even in the pure limit[1] (i.e., $l/\xi_0 \gg 1$, where l is the electric mean free path and ξ_0 is the BCS coherence length), which cannot be understood in the above theory.

In a previous paper[12] (we refer to it hereafter as I) we were able to circumvent the difficulty associated with the power expansion in Δ, and to calculate various transport properties by making use of a Green's function due to Brandt, Pesch, and Tewordt[13,14] (BPT). However, we considered only those transport coefficients in which the contribution from the dynamical fluctuation of the order parameter could be completely neglected.[15] In this paper we calculate those transport coefficients of a clean type-II superconductor, in which the dynamical fluctuation plays the central role. Making use of the formalism developed by Caroli and Maki,[15] we can express the transport coefficients in terms of retarded products, which are taken in the state without fluctuation. The retarded products are obtained by making use of the BPT Green's function, which should be exact if we are close enough to the upper critical field. In Sec. II we summarize the formalism necessary to calculate the flux-flow conductivity and the Ettingshausen effect. The relevant retarded products are computed analytically for small T (i.e., $T \cong 0$ °K) in Sec. III. If we expand the resulting expression in powers of Δ, and compare with the previous calculation,[7] we find that the Ettingshausen coefficient is the same in both calculations, while the flux-flow resistivity is larger, by a factor of 2, than the previous result. This factor of 2 may be of some importance in improving the agreement between theory and experiment. The present calculation also shows that higher-order terms in Δ become appreciable if l/ξ_0 is large, and therefore the expansion in powers of Δ has only a limited range of validity.

II. FORMULATION

Let us consider a clean type-II superconductor in the vortex state. A magnetic field H slightly smaller than H_{c2} is applied along the z axis, and an electric field E is applied along the x direction. In terms of linear response theory, which takes into account the fluctuations of the order parameter,[15] the flux-flow conductivity and the Ettingshausen coefficient are determined by

$$\sigma_{xx} = \lim_{\omega \to 0} \left(\frac{Q_{j_x j_x}(-i\omega)}{-i\omega} \right) , \tag{1}$$

$$\alpha_{yx} = \lim_{\omega \to 0} \left(\frac{Q_{j_y^h j_x}(-i\omega)}{-i\omega} \right) , \tag{2}$$

where

$$Q_{\vec{j}\,\vec{j}}(-i\omega) = \langle [\vec{j}, \vec{j}\,] \rangle (-i\omega) + \langle [\vec{j}, \Psi^\dagger] \rangle D_1 \langle [\Psi, \vec{j}\,] \rangle (-i\omega) + \langle [\vec{j}, \Psi] \rangle D_1 \langle [\Psi^\dagger, \vec{j}\,] \rangle (-i\omega) , \tag{3}$$

$$Q_{\vec{j}^h \vec{j}}(-i\omega) = \langle [\vec{j}^h, \vec{j}\,] \rangle (-i\omega) + \langle [\vec{j}^h, \Psi^\dagger] \rangle D_1 \langle [\Psi, \vec{j}\,] \rangle (-i\omega) + \langle [\vec{j}^h, \Psi] \rangle D_1 \langle [\Psi^\dagger, \vec{j}\,] \rangle (-i\omega) , \tag{4}$$

$$D_1(-i\omega) = \frac{|g|}{1 - |g| \langle [\Psi, \Psi^\dagger] \rangle (-i\omega)} . \tag{5}$$

Here, the subscript 1 on D indicates the fluctuation with azimuthal quantum number $n = 1$, since the current operator only couples[15] the equilibrium order parameter Δ_0 (which corresponds to a state with $n = 0$) to the fluctuation with $n = 1$. All the retarded products in Eqs. (3)–(5) are to be taken in the vortex state with fluctuation.[15]

Since we are interested here in the transport properties in a dc electric field, we can expand the retarded products in powers of ω, the external frequency. Retaining the term of the order ω, we can further simplify Eqs. (1) and (2) as

$$\sigma_{xx} = \sigma^{st} + \sigma^{fl} , \tag{6}$$

$$\alpha = \alpha^{fl} , \tag{7}$$

where

$$\sigma^{st} = A_{j_x j_x} , \tag{8}$$

$$\sigma^{fl} = 4A_{j\Psi} \{ D_1(0) \langle [\Psi, \vec{j}\,] \rangle (0) \} , \tag{9}$$

$$\alpha^{fl} = 2A_{j^h\Psi} \{ D_1(0) \langle [\Psi, j] \rangle (0) \} . \tag{10}$$

Here we have made use of the expansions

$$\langle [\vec{j}, \vec{j}\,] \rangle (-i\omega) = \langle [\vec{j}, \vec{j}\,] \rangle (0) - i\omega A_{\vec{j}\,\vec{j}} + O(\omega^2) , \tag{11}$$

$$\langle[\vec{j}, \Psi]\rangle(-i\omega) = \langle[\Psi, \vec{j}]\rangle(-i\omega)$$
$$= \langle[\Psi^\dagger, \vec{j}]\rangle(-i\omega) = \langle[j, \Psi^\dagger]\rangle(-i\omega)$$
$$= \langle[j, \Psi]\rangle(0) - i\omega A_{\vec{j}\Psi} + O(\omega^2) , \quad (12)$$

$$\langle[j^h, \Psi]\rangle(-i\omega) = \langle[j^h, \Psi^\dagger]\rangle(-i\omega) = -i\omega A_{j^h\Psi} + O(\omega^2) , \quad (13)$$

where[15]

$$\langle[\vec{j}^h, \Psi]\rangle(0) \equiv 0 .$$

In addition, in the above reduction we have neglected the contribution containing $D_1^{(1)}$, the coefficient of ω in $D_1(-i\omega)$. We have

$$D_1(-i\omega) = D_1(0) - i\omega D_1^{(1)} , \quad (14)$$

since $D_1^{(1)}$ is smaller than $A_{\vec{j}\Psi}$ (or $A_{\vec{j}^h\Psi}$) by a factor ξ_0/l.

In the following we also make use of the previous results of Caroli and Maki[15] (CM) to calculate $D_1(0)\langle[\Psi, \vec{j}]\rangle(0)$, since in the calculation of static properties, we can make use of the power expansion in Δ without any difficulty. Therefore, our task reduced to the calculation of the imaginary parts of the retarded products $A_{\vec{j}\vec{j}}$, $A_{\vec{j}\Psi}$, and $A_{\vec{j}^h\Psi}$. In closing this section we note that there is no contribution to the Ettingshausen coefficient (or to the thermoelectric power) from $\langle[\vec{j}^h, \vec{j}]\rangle$, as it vanishes identically due to the symmetry of the quasiparticle and hole spectrum.

Furthermore, for the thermoelectric power we have

$$\langle[j_x^h, \Psi]\rangle(-i\omega) = -\langle[j_x^h, \Psi^\dagger]\rangle(-i\omega) , \quad (15)$$

and the contribution from the fluctuation cancels exactly as in the case of a dirty type-II superconductor.[3,5,6]

III. FLUX-FLOW CONDUCTIVITY AND ETTINGSHAUSEN EFFECT

A. Flux-Flow Resistivity

In order to obtain the flux-flow conductivity, as we have already shown in Sec. II, it is only necessary to calculate $A_{\vec{j}\Psi}$ and $A_{\vec{j}\vec{j}}$. We first consider the nonfluctuation part of the dc conductivity. The

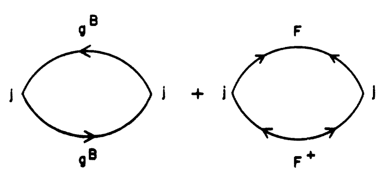

FIG. 1. Diagrams contributing to σ^{st} are shown. Here g^B denotes the (BPT) Green's function [see Eq. (17)].

calculation proceeds in exactly the same way as the previous calculation of the thermal conductivity and the ultrasonic attenuation.[12] $A_{\vec{j}\vec{j}}$ can be expressed in terms of the BPT Green's function[13] as (see Fig. 1)

$$A_{\vec{j}\vec{j}} = \frac{\sigma}{2\pi}\int \frac{d\Omega}{4\pi}\,\frac{2}{3}\sin^2\theta \int \frac{d\omega}{2T}\operatorname{sech}^2\left(\frac{\omega}{2T}\right) \times \left(G^B(\vec{p}, \omega+i\delta)\,G^B(\vec{p}, \omega-i\delta) + \Delta^2\int_{-\infty}^{\infty}\rho(u)\,du\,F(\vec{p}, u, \omega+i\delta)\,F(\vec{p}, u, \omega-i\delta)\right) , \quad (16)$$

where

$$G^B(\vec{p}, i\omega_n) = \left(i\tilde{\omega}_n - \xi_p - \Delta^2\int_{-\infty}^{\infty}\frac{\rho(u)\,du}{i\tilde{\omega}_n + \xi_p - u}\right)^{-1} , \quad (17)$$

$$F(\vec{p}, u, i\omega_n) = G^B(\vec{p}, i\omega_n)\,(i\tilde{\omega}_n + \xi_p - u)^{-1} , \quad (18)$$

$$\rho(u, \Omega) = \frac{1}{\sqrt{\pi}}\,\frac{e^{-\mu^2/(k_c v_F \sin\theta)^2}}{(k_c v_F \sin\theta)^2} , \quad (19)$$

$$k_c = (2eH_{c2})^{-1/2}, \qquad \tilde{\omega}_n = \omega_n\left(1 + \frac{1}{2\tau|\omega_n|}\right),$$

$$\xi_p = \frac{p^2}{2m} - \mu ,$$

where τ is the electronic lifetime, and θ is the angle between the dc magnetic field and the momentum $\vec{p}$ of the electron. Following the procedure used in I, we can transform Eq. (14) at low temperatures (i.e., $T \cong 0$ °K) into

$$A_{\vec{j}\vec{j}} = \tfrac{3}{2}\sigma\int_0^1 dz\,(1-z^2)\,\frac{1}{1 - i\sqrt{\pi}\,\Delta^2 W'(ix_0)} \times \frac{1 + \mu/(1-z^2)^{1/2}}{1 + \mu/(1-z^2)^{1/2}} , \quad (20)$$

where

$$W(z) = \frac{i}{\pi}\int \frac{e^{-t^2}}{z - t}\,dt , \quad (21)$$

$\mu = 2\sqrt{\pi}\,\tau\Delta^2/k_c v_F$, and x_0 is the root of the equation

$$x_0 = 1/\tau + \Delta^2\sqrt{\pi}\,W(i(x_0 - 1/\tau)) . \quad (22)$$

In Eqs. (20) and (22) we measure energy in units of $k_c v_F \sin\theta$. If we now substitute the approximate relation

$$\frac{1}{1 - i\sqrt{\pi}\,\Delta^2 W'(ix_0)} \cong \frac{1}{1 - 2x_0^2 + 2\Delta^2} = 1 - \frac{2\Delta^2(\alpha + 2\Delta^2)}{\alpha + 2\Delta^2(2+\alpha) + 4\Delta^4} \quad (23)$$

into Eq. (20), returning to natural units [i.e., replace Δ by $\Delta/k_c v_F(1-z^2)^{1/2}$], we have

$$\sigma^{st}=\sigma\left[1-3\alpha\left(\frac{\Delta}{k_c v_F}\right)^2\int_0^1 dz\,\frac{1+(2/\alpha)(\Delta/k_c v_F)^2(1-z^2)^{-1}}{\alpha+2(2+\alpha)(\Delta/k_c v_F)^2(1-z^2)^{-1}+4(\Delta/k_c v_F)^4(1-z^2)^{-2}}\right]$$

$$=\sigma\left\{1-3\left(\frac{\Delta}{k_c v_F}\right)^2\left[1-\left(\frac{1}{\alpha}+1\right)\left(\frac{\Delta}{k_c v_F}\right)^2\ln\left(\frac{2\alpha}{2+\alpha}\left(\frac{k_c v_F}{\Delta}\right)\right)^2\right]\right\}, \tag{24}$$

where $\alpha=4/\pi$.

We see that the μ-dependent terms in A_{jj} cancel exactly, and σ can be well expressed by a first correction [of the order $(\Delta/\epsilon)^2$] which is independent of the electronic mean free path. It is worthy of note that σ^{st} decreases, instead of increasing, in the vortex state as the magnetic field becomes smaller.

Now let us turn to the fluctuation contribution σ^{fl}. As already stated in Sec. II, we calculate this contribution in two steps. First we note that for $D_1(0)\langle[\vec{j},\Psi^\dagger]\rangle(0)$ we can make use of the CM result with a good approximation. The corresponding expression in CM yields[16]

$$\delta\Delta_1^\dagger=D_1(0)\langle[\vec{j},\Psi^\dagger]\rangle(0)\,\delta\vec{A}$$

$$=\frac{ev_F}{\sqrt{2}\,\epsilon_0}\left(\frac{\Pi_+}{(4eH)^{1/2}}\Delta_0\right)\delta\vec{A}\ , \tag{25}$$

where $\epsilon_0=\frac{1}{2}k_c v_F$, $\vec{\Pi}=\vec{\nabla}/i-2e\vec{A}$, and Δ_0 is the order parameter for the vortex state describing the triangular Abrikosov structure. Making use of this $\delta\Delta_1^\dagger$, the diagram we calculate is given, as shown in Fig. 2, by

$$A_{j\Psi}=\frac{2eN(0)\Delta_0}{2\pi}\int\frac{d\Omega}{4\pi}v_\mu\int\frac{d\omega}{2T}\operatorname{sech}^{-2}\left(\frac{\omega}{2T}\right)\int d\xi\left(G^B(\vec{p},\omega+i\delta)\int d\alpha\,\rho_{10}(\alpha)F(\vec{p},2\alpha,\omega-i\delta)\right)$$

$$=2eN(0)\Delta_0\int\frac{d\Omega}{4\pi}v_\mu\int\frac{d\omega}{2T}\operatorname{sech}^{-2}\left(\frac{\omega}{2T}\right)I(\omega,\Omega)\ , \tag{26}$$

where

$$I(\omega,\Omega)=\tfrac{1}{2}\pi\int d\xi\,G^B(\vec{p},\omega+i\delta)\int d\alpha\,\rho_{10}(\alpha)F(\vec{p},2\alpha,\omega-i\delta)$$

$$=\frac{1}{2\pi(k_c v_F\sin\theta)^2}\left(\frac{2}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}du\,(-ui)\,e^{-i\phi-u^2}\int_{-\infty}^{\infty}d\xi\,\frac{1}{\xi-\omega-i/2\tau-i\sqrt{\pi}\,\Delta^2W(\xi+\omega+i/2\tau)}$$

$$\times\frac{1}{[\xi-\omega+i/2\tau-i\sqrt{\pi}\,\Delta^2W(\xi+\omega-i/2\tau)](\xi+\omega+i/2\tau-u)} \tag{27}$$

and $\rho_{10}(\alpha)$, already defined by CM, is given by

$$\rho_{10}(\alpha,\Omega)=2\langle\phi_0^*(\vec{q})\phi_1(\vec{q})\delta(\vec{q}\cdot\vec{v}-2\alpha)\rangle=-i\alpha\left(\frac{2}{\pi}\right)^{1/2}\frac{e^{-i\phi}}{(\epsilon_0\sin\theta)^2}e^{-(\alpha/\epsilon_0\sin\theta)^2}\ . \tag{28}$$

Equation (27) can be transformed into

$$I(\omega,\Omega)=\frac{-i}{(k_c v_F\sin\theta)^2}\left(\frac{2}{\pi}\right)^{1/2}\frac{e^{-i\phi}}{4\pi}\int\frac{d\xi\,\pi W'(\xi+\omega+i/2\tau)}{\xi-\omega-i/2\tau-i\sqrt{\pi}\,\Delta^2W(\xi+\omega+i/2\tau)}$$

$$\times\frac{1}{\xi-\omega+i/2\tau-i\sqrt{\pi}\,\Delta^2W(\xi+\omega+i/2\tau)}\ , \tag{29}$$

where we have made use of the relation

$$\int_{-\infty}^{\infty}(-iu)\frac{e^{-u^2}du}{\omega+\frac{1}{2}\tau+\xi-u}=\frac{i}{2}\int_{-\infty}^{\infty}\left(\frac{\partial}{\partial u}e^{-u^2}\right)\frac{du}{\omega+i/2\tau+\xi-u}=\tfrac{1}{2}\pi W'(\omega+i/2\tau+\xi) \tag{30}$$

and $W(z)$ is already defined in Eq. (21). Finally, closing the path of integration in the upper half-plane, we evaluate the integral

$$I(\omega,\Omega)=\left(\frac{\pi}{2}\right)^{1/2}\frac{e^{-i\phi}}{(k_c v_F\sin\theta)^2}\frac{-iW'(iz_0)}{[1-i\sqrt{\pi}\,\Delta^2W'(iz_0)][z_0+2\omega i-\sqrt{\pi}\,\Delta^2W_I(iz_0)]}\ , \tag{31}$$

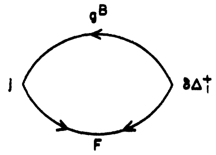

FIG. 2. Diagram contributing to σ^{fl}, which involves the dynamical fluctuation of the order parameter.

where

$$W_I(iz_0) = W(iz_0) - 2e^{z_0^2} \tag{32}$$

[i.e., $W_I(iz_0)$ is the analytical continuation of $W(iz_0)$ into the second Riemann sheet (cf. I)] and z_0 is the root of the following equation:

$$z_0 = 1/\tau - 2i\omega + \sqrt{\pi}\,\Delta^2 W(iz_0)\ . \tag{33}$$

In all the above expressions (27), (29)–(31), and (33), we have measured energy in units of $k_c v_F \sin\theta$.

In order to calculate the electric conductivity at $T \cong 0$ °K we need only $I(0,\Omega)$ [i.e., $I(\omega,\Omega)$ for $\omega = 0$]. However, in our subsequent calculation of the Ettingshausen coefficient we need the same function for small (nonvanishing) ω. Therefore we shall calculate Eq. (31) for small but nonvanishing ω (i.e., we assume $\omega/\epsilon \ll 1$). (Note this expansion may also be used to calculate the electric conductivity at finite temperature if $T/\epsilon \ll 1$.) For this purpose we will make use of the asymptotic expansion of $W(iz_0)$, which is valid if $|z_0| \ll 1$:

$$W(iz_0) = e^{z_0^2} - (2/\sqrt{\pi})z_0[1 + O(z_0^2)]\ . \tag{34}$$

Substituting Eq. (34) into Eq. (33), we can solve for z_0 by iteration as

$$z_0 = 1/\tau - 2\omega i + \sqrt{\pi}\,\Delta^2[e^{-4\omega^2} - (2/\sqrt{\pi})(1/\tau - 2\omega i + \sqrt{\pi}\,\Delta^2)]\ , \tag{35}$$

and $I(\omega,\Omega)$ can be expressed as

$$I(\omega,\Omega) = \left(\frac{\pi}{2}\right)^{1/2} \frac{e^{-i\phi}}{(k_c v_F \sin\theta)^2}\, \frac{2/\sqrt{\pi} + 4i\omega e^{-4\omega^2}}{1/\tau + 2\sqrt{\pi}\,\Delta^2 e^{-4\omega^2}}\ . \tag{36}$$

Finally, $A_{\vec{j}\Psi}$ is obtained as (we return to natural units)

$$A_{\vec{j}\Psi} = \frac{4eN(0)\Delta_0}{\sqrt{2}} \int \frac{d\Omega}{4\pi} v_x \frac{e^{-i\phi}}{(k_c v_F \sin\theta)^2[1/(k_c l \sin\theta) + 2\sqrt{\pi}\Delta^2/(k_c v_F \sin\theta)^2]}$$
$$= \frac{\sqrt{2}\, e\tau N(0) v_F \Delta}{k_c v_F} \int_0^1 \frac{dz\,(1-z^2)^{1/2}}{(1-z^2)^{1/2} + \mu}\ , \tag{37}$$

where μ has already been defined just after Eq. (21).

The fluctuation contribution to the dc conductivity is then given by

$$\sigma^{fl} = 4A_{\vec{j}\Psi^\dagger} D_1(0)\langle[\Psi,\vec{j}\,]\rangle(0) = 4\left(\frac{e^2 v_F^2 N(0)\tau}{\epsilon_0 k_c v_F}\right)\int_0^1 \frac{dz\,(1-z^2)^{1/2}}{(1-z^2)^{1/2}+\mu}$$
$$= 12\sigma(\Delta/k_c v_F)^2 F_1(\mu)\ , \tag{38}$$

where

$$F_1(\mu) = \int_0^1 \frac{dz\,(1-z^2)^{1/2}}{(1-z^2)^{1/2}+\mu} = 1 - \mu\left[\frac{\pi}{2} - \frac{2\mu}{(1-\mu^2)^{1/2}}\operatorname{arctanh}\left(\frac{1-\mu}{1+\mu}\right)\right] \quad \text{for } \mu < 1$$
$$= 1 - \mu\left[\frac{\pi}{2} - \frac{2\mu}{(\mu^2-1)^{1/2}}\operatorname{actan}\left(\frac{\mu-1}{\mu+1}\right)\right] \quad \text{for } \mu > 1\ . \tag{39}$$

The total dc conductivity in the flux-flow regime is given by

$$\sigma_s = \sigma[1 - 3(\Delta/k_c v_F)^2 + 12(\Delta/k_c v_F)^2 F_1(\mu)]\ . \tag{40}$$

The last term in Eq. (40) can be compared with the previous calculation by one of us[7] (K. M.); it is found to be larger by a factor of 2. This difference originates from the fact that in the previous calculation only a part of the diagram was taken into account.

The present calculation indicates that σ has a somewhat stronger dependence on $|\Delta|^2$, which improves significantly the agreement with experiment. Furthermore σ^{fl} has a strong mean-free-path dependence through $\mu = 2\sqrt{\pi}\,(\tau\Delta^2/k_c v_F)$, although the slope of the resistivity, $(H/R)\,\partial R/\partial H|_{H=H_{c2}}$, at $H = H_{c2}$ is independent of l in the pure limit. In Fig. 3 the field dependence of the flux-flow resistivity $R(H) = \sigma_s^{-1}$ is plotted for several values of l/ξ_0. In the case $2\sqrt{\pi}k_c l = 20$ the resistivity increases as Δ^2 increases for $(\Delta/k_c v_F)^2 > 0.02$. We believe that this unphysical behavior is due to the fact that our approximation is still incapable of dealing with the extremely pure superconductor.

B. Ettingshausen Effect

In the Ettingshausen effect we have only contributions from the fluctuation term. As in the case of the electric conductivity the problem reduces to the calculation of the diagram given in Fig. 2 where $\vec{j}$ is now replaced by $\vec{j}^h$, the heat current:

$$A_{j^h_\mu \Psi} = \frac{2eN(0)\Delta_0}{2\pi}\int \frac{d\Omega}{4\pi}\, v_\mu \int \frac{d\omega}{2T}\,\omega\,\mathrm{sech}^2\left(\frac{\omega}{2T}\right)\int d\xi \left(G^B(\vec{p},\omega+i\delta)\int d\alpha\,\rho_{10}(\alpha)\,F(\vec{p},2\alpha,\omega-i\delta)\right)$$

$$= 2eN(0)\Delta_0 \int \frac{d\Omega}{4\pi}\, 2v_\mu \int \mathrm{sech}^2\left(\frac{\omega}{2T}\right)\frac{\omega^2}{2T}\, I(\omega,\Omega)\, d\omega$$

$$= 2eN(0)\Delta_0\, v_F\,\tau\left(\frac{\pi}{2}\right)^{1/2}\frac{\langle\omega^2\rangle}{(k_c v_F)^2}\int_0^1 \frac{dz}{(1-z^2)^{1/2}+\mu}\,, \qquad (41)$$

where

$$\langle\omega^2\rangle = \int_0^\infty \frac{d\omega}{2T}\,\omega^2\,\mathrm{sech}^{-2}\left(\frac{\omega}{2T}\right) = \tfrac{1}{3}(\pi T)^2\,. \qquad (42)$$

In the derivation of Eq. (41) we made use of $I(\omega,\Omega)$ obtained in Eq. (36). Combining this with the expression $D_1^{(0)}\langle[\Psi, j\,]\rangle(0)$ defined in Eq. (25), we have

$$\alpha = \alpha^{t1} = -2\,(2\pi)^{1/2}\,\epsilon^2 N(0)\,\frac{lv\langle\omega^2\rangle|\Delta|^2}{(k_c v_F)^2}\,\frac{\sqrt{2}}{\epsilon_0} \times \int_0^1 dz\,\frac{1}{(1-z^2)^{1/2}+\mu}$$

$$= -12\sqrt{\pi}\,\frac{\sigma}{e}\,\frac{|\Delta|^2}{k_c v_F}\,\frac{\langle\omega^2\rangle}{k_c v_F}\int_0^1 dz\,\frac{1}{(1-z^2)^{1/2}+\mu} \qquad (43)$$

$$= -6\,\frac{eN}{m}\,\frac{\langle\omega^2\rangle}{(k_c v_F)^2}\,\mu\int_0^1 dz\,\frac{1}{(1-z^2)^{1/2}+\mu}$$

$$= -6\,\frac{eN}{m}\,\frac{\langle\omega^2\rangle}{(k_c v_F)^2}\,F_2(\mu)\,, \qquad (44)$$

where

$$F_2(\mu) = \mu\left[\frac{\pi}{2} - \frac{2\mu}{(1-\mu^2)^{1/2}}\,\mathrm{arctanh}\left(\frac{1-\mu}{1+\mu}\right)^{1/2}\right] \quad \text{for } \mu < 1$$

$$= \mu\left[\frac{\pi}{2} - \frac{2\mu}{(\mu^2-1)^{1/2}}\,\arctan\left(\frac{\mu-1}{\mu+1}\right)^{1/2}\right] \quad \text{for } \mu > 1\,. \qquad (45)$$

This expression is equivalent to the previous calculation (K. M.) in the limit μ tends to zero. This follows from the fact that as $\langle[y^h, \Psi]\rangle(0) \equiv 0$ in the calculation of the Ettingshausen coefficient we need only the static $\delta\Delta$, which is correctly given by the previous procedure (K. M.). For a clean type-II superconductor, the Ettingshausen coefficient at low temperature can be expressed in terms

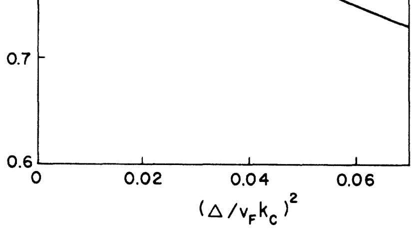

FIG. 3. Reduced resistivity $R(H)/R$ in the flux-flow regime is drawn as functions of $(\Delta/v_F k_c)^2$ for several values of $2\sqrt{\pi}lk_c$ $(\cong l/\xi_0)$.

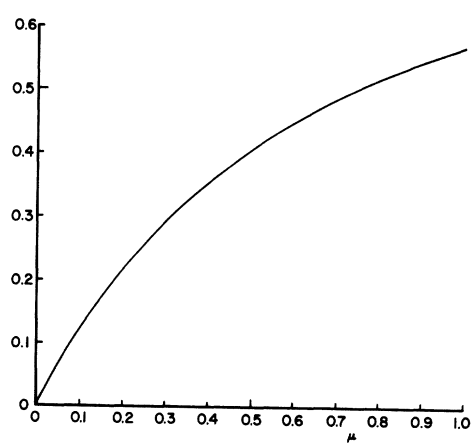

FIG. 4. Universal function $F_2(\mu)$, which appears in the expression for the delivered entropy per flux line, is shown as a function of μ.

of a universal function $F_2(\mu)$ of (μ); this explains qualitatively the magnetic-field dependence observed by Serin and Fiory[1] in two Nb samples with different purity.

Making use of α, we can also calculate S_D, the delivered entropy per unit flux, from Eq. (43). S_D is defined by

$$S_D = \frac{-\pi}{eT}\,\alpha = \frac{2\pi^3 N}{mT}\left(\frac{T}{k_c v_F}\right)^2 \mu \int_0^1 \frac{dz}{(1-z^2)^{1/2}+\mu} \,. \tag{46}$$

The quantity

$$S_D \Big/ \left(\frac{2\pi^3}{m} N \frac{T}{(k_c v_F)}\,\alpha\right) = F_2(\mu)$$

is plotted in Fig. 4.

In addition to the Ettingshausen coefficient, we can also express the Nernst coefficient in terms of the α given above. However, in order to determine the Peltier coefficient it is necessary to calculate the Hall current, which is outside the scope of the present treatment.

IV. CONCLUDING REMARK

Making use of the (BPT) Green's function, we recalculate the flux-flow resistivity and the Ettingshausen effect for clean type-II superconductors at low temperatures. The present results seem to improve greatly agreement with existing experimental data, although more detailed comparisons are certainly desirable. We also show that the transport coefficients depend strongly on mean free path even in the clean limit, although the slope of various quantities at $H = H_{c2}$ is independent of l. In the present formulation we are still unable to handle the Hall effect and the related properties (e.g., the Peltier effect) in the flux-flow regime, as we did not take into account higher-order terms in $(lp_0)^{-1}$.

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